EXERCISE 2(d)

Q2.) Using Quine-Mccluskey Tabular Method simplify the following Boolean expressions.

i) $F(W,X,Y,Z) = \sum m(0,4,5,7,8,11,12,15)$

WX \ YZ	00	01	11	10
00	m_0 (0)	m_1	m_3	m_2
01	m_4 (1)	m_5 (2)	m_7 (3)	m_6
11	m_{12} (2)	m_{13}	m_{15} (4)	m_{14}
10	m_8 (1)	m_9	m_{11} (3)	m_{10}

(?) in the K-table shows no: of 1's in each min term.

The given function is having 4 variables W, X, Y & Z.
The given minterms are 0, 4, 5, 7, 8, 11, 12 and 15. No: of 1's in these min terms are 0, 1, 2, 3, 1, 3, 2, 4 respectively

∴ The ascending order of these min terms based on the number of ones present in their binary equivalent is

0, 4, 8, 5, 12, 7, 11, 15.

The following table shows these minterms and their equivalent binary representation.

Group Name	Min Terms	W	X	Y	Z
GA I	0	0	0	0	0 ✓
	4	0	1	0	0 ✓
GB II	8	1	0	0	0 ✓
GB III	5	0	1	0	1 ✓
	12	1	1	0	0 ✓
GB IV	7	0	1	1	1 ✓
	11	1	0	1	1 ✓
GB V	15	1	1	1	1 ✓

The following table shows the possible merging of minterms from the adjacent grops. The minterms, which are differed in only one bit position from the adjacent group are merged and differed bit is represented by the symbol '—' in the table. The remaining bits are kept as they are.

Group Name	Minterms	W	X	Y	Z	
GBI	0, 4	0	—	0	0 ✓	Prime Implicants
	0, 8	—	0	0	0 ✓	
	4, 5	0	1	0	— X	W'XY' (not merged)
GB II	4, 12	—	1	0	0 ✓	
	8, 12	1	—	0	0 ✓	
GB III	5, 7	0	1	—	1 X	W'XZ (not maged)
GB IV	7, 15	—	1	1	1 X	XYZ (")
	11, 15	1	—	1	1 X	WYZ (")

Group Name	Minterm	W	X	Y	Z	
GBI	0, 4, 8, 12	—	—	0	0	Y'Z'
	0, 8, 4, 12	—	—	0	0	
GB II						

Min Terms / Prime Implicants	0	4	5	7	8	11	12	15
Y'Z'	1	1			1		1	
W'XY'		1	1					
WXZ			1	1				
XYZ				1				1
WYZ						1		1

EXERCISE 2(d)

Q2. (ii) $f(A,B,C,D) = \sum m(0,1,2,3,5,7,8,10,12,13,15)$

	0000	0001	0010	0011	0101	0111	1000	1010	1100	1101	1111
No of ones	0	1	1	2	2	3	1	2	2	3	4

The ascending order of minterms based on number of 1's in their binary representations is 0, 1, 2, 8, 3, 5, 10, 12, 7, 13, 15.

Group	Minterms	A B C D
I	m_0	0 0 0 0 ✓
	m_1	0 0 0 1 ✓
II	m_2	0 0 1 0 ✓
	m_8	1 0 0 0 ✓
	m_3	0 0 1 1 ✓
III	m_5	0 1 0 1 ✓
	m_{10}	1 0 1 0 ✓
	m_{12}	1 1 0 0 ✓
	m_7	0 1 1 1
IV	m_{13}	1 1 0 1
	m_{15}	1 1 1 1
V		

Group	Merged Minterms	A B C D
I	0, 1	0 0 0 − ✓
	0, 2	0 0 − 0 ✓
	0, 8	− 0 0 0 ✓
II	1, 3	0 0 − 1 ✓
	1, 5	0 − 0 1 ✗
	2, 3	0 0 1 − ✓
	2, 10	− 0 1 0 ✓
	8, 10	1 0 − 0 ✓
	8, 12	1 − 0 0 (X) AC'D'
III	3, 7	0 − 1 1 ✓
	5, 7	0 1 − 1 ✓
	5, 13	− 1 0 1 ✓
	12, 13	1 1 0 − X ABC'
IV	7, 15	− 1 1 1 ✓
	13, 15	1 1 − 1 ✓

Group	Merged Minterms	A B C D	
I	0, 1, 2, 3	0 0 − −	AB
	0, 2, 1, 3	0 0 − −	
	0, 2, 8, 10	− 0 − 0	B'D'
	0, 8, 2, 10	− 0 − 0	
II	1, 3, 5, 7	0 − − 1	A'D
	1, 5, 3, 7	0 − − 1	
III	5, 7, 13, 15	− 1 − 1	BD
	5, 13, 7, 15	− 1 − 1	

Prime implicant table is shown below:

Minterms/ Prime Implicants	0	1	2	3	5	7	8	10	12	13	15
A'B'	X	X	X	X							
B'D'	X		X				X	(X)			
A'D		X		X	X	X					
BD					X	X				X	(X)
AC'D'							X		X		
ABC'									X	X	

The minterm m_{10} is singly covered by the prime implicant B'D'. Therefore B'D' is an essential prime implicant. Also minterm m_{15} is covered by a single prime implicant (BD). Therefore BD is also an essential prime implicant. So B'D' and BD are part of simplified Boolean function.

Now remove the rows containing B'D' and BD and their corresponding minterms (ie 0, 2, 5, 7, 8, 10, 13, 15)

The reduced prime implicant table is shown below.

Minterms/Prime Implicants	1	3	12
A'B'	X	X	
A'D	X	X	
AC'D'			X
ABC'			X

Now prime implicant A'B' covers both the minterms 1 & 3 & so do the prime implicant A'D. ∴ we can take one

of them in the simplified Boolean function. $AC'D'$ covers the minterm m_{12}. Similarly ABC' also covers m_{12}. ∴ either of $AC'D'$ & ABC' can be taken in simplified function.

So simplified form of given function

$$f(A,B,C,D) = B'D' + BD + A'B' + AC'D'$$
$$\text{or } B'D' + BD + A'D + AC'D'$$
$$\text{or } B'D' + BD + A'B' + ABC'$$
$$\text{or } B'D' + BD + A'D + ABC'$$

(3) $f(A,B,C,D) = \sum m(3,7,11,12,13,14,15)$

0011 0111 1011 1100 1101 1110 1111

No: of 1's → 2, 3, 3, 2, 3, 3, 4

The ascending order of these minterms based on number of 1's in their binary equivalent is 3, 12, 7, 11, 13, 14, 15

The following table shows these minterms, their merger in step II & Step III

← Step I → ← Step II → ← Step III

Group	Min terms	A B C D	Group	Merged minterms	A B C D	Group	Merged minterms	A B C D	
I	3	0 0 1 1 ✓		3, 7	0 − 1 1 ✓		3, 7, 11, 15	− − 1 1	CD
	12	1 1 0 0 ✓		3, 11	− 0 1 1 ✓		3, 11, 7, 15	− − 1 1	
	7	0 1 1 1 ✓	I	12, 13	1 1 0 − ✓	I	12, 13, 14, 15	1 1 − −	AB
II	11	1 0 1 1 ✓		12, 14	1 1 − 0 ✓		12, 14, 13, 15	1 1 − −	
	13	1 1 0 1 ✓		7, 15	− 1 1 1 ✓				
	14	1 1 1 0 ✓	II	11, 15	1 − 1 1 ✓				
III	15	1 1 1 1 ✓		13, 15	1 1 − 1 ✓				
				14, 15	1 1 1 − ✓				

The Prime implecant table is shown below

Minterms / Prime implicant	3	7	11	12	13	14	15
CD	X	X	X				X
AB				X	X	X	X

The minterms 3, 7, 11 are Covered by one prime implicant CD & minterms 12, 13, 14 are Covered by only prime implicant AB.

∴ Simplified Boolean function is $f(A,B,C,D) = AB + CD$. Ans